

**LO.a: calculate and evaluate the predicted trend value for a time series, modeled as either a linear trend or a log-linear trend, given the estimated trend coefficients.**

1. For a log linear trend model, the slope coefficient is 5 and the intercept coefficient is 10. The predicted trend value at time  $t = 20$  is closest to:
  - A. 110.
  - B. 5.92
  - C. 62.34

**LO.b: describe factors that determine whether a linear or a log-linear trend should be used with a particular time series and evaluate limitations of trend models.**

2. In which of the following scenarios will a log-linear trend model be *most likely* used?
  - A. When the variable grows at a constant rate.
  - B. When the variable increases over time by a constant amount.
  - C. The Durbin Watson statistic is significantly different from 2.0.

**LO.c: Explain the requirement for a time series to be covariance stationary and describe the significance of a series that is not stationary.**

3. Which of the following is *least likely* a requirement for a time series to be covariance stationary?
  - A. The expected value of the time series changes at a constant rate over time.
  - B. The time series' volatility around its mean does not change over time.
  - C. The covariance of the time series with leading or lagged values of itself is constant.

**LO.d: Describe the structure of an autoregressive (AR) model of order  $p$  and calculate one- and two-period-ahead forecasts given the estimated coefficients.**

4. Consider an AR(1) model with the following equation  $x_t = 2.5 + 0.2x_{t-1}$ . If the current value of  $x$  is 4, the two-period-ahead forecast is *closest* to:
  - A. 2.5
  - B. 3.3
  - C. 3.16

**LO.e: Explain how autocorrelations of the residuals can be used to test whether the autoregressive model fits the time series.**

5. The correlations of the error terms from the estimation of an AR(1) model using a sample with 144 observations is presented in the following figure. Based on this information which of the following statements is *most appropriate*?  
(The critical two tail t-value at the 5% significance level and 142 degrees of freedom is 1.98)

Lag	Autocorrelation
1	0.124
2	0.148
3	0.166

- A. The AR model is correctly specified.
- B. The AR model is not specified correctly because the autocorrelations of the residuals for lag 2 are statistically different from 0.
- C. The AR model is not specified correctly because the autocorrelations of the residuals for lag 3 are statistically different from 0.

**LO.f: Explain mean reversion and calculate a mean-reverting level.**

6. For a regression model  $x_t = 2.5 + 0.2x_{t-1}$ , the mean reverting level is *closest* to:
- A. 12.5.
  - B. 3.125.
  - C. 4.65.

**LO.g: Contrast in-sample and out-of-sample forecasts and compare the forecasting accuracy of different time-series models based on the root mean squared error criterion.**

7. Which of the following statements is *most accurate*? When comparing two autoregressive models:
- A. the model with the higher root mean squared error (RMSE) for out-of-sample data is expected to produce better predictive power in the future.
  - B. the model with the lower root mean squared error (RMSE) for in-sample data is expected to produce better predictive power in the future.
  - C. the model with the lower root mean squared error (RMSE) for out-of-sample data is expected to produce the better predictive power in the future.

**LO.h: Explain the instability of coefficients of time-series models.**

8. Analyst 1: The coefficients of models estimated with shorter time series are usually less stable than those with longer time series.  
Analyst 2: If there has been a dramatic change in the underlying economic environment, then historical data may not provide a reliable model.
- A. Analyst 1 is correct.
  - B. Analyst 2 is correct.
  - C. Both analysts are correct.

**LO.i: Describe characteristics of random walk processes and contrast them to covariance stationary processes.**

9. A time series that follows a random walk process, has the following expression  $x_t = b_0 + b_1x_{t-1} + \epsilon_t$ . Which of the following statements is *least accurate*?
- A. The expected value of the error term  $E(\epsilon_t)$  is 0 for a random walk with or without drift.
  - B.  $b_0 = 0$  for a random walk without drift and  $b_0 \neq 0$  for a random walk with drift.
  - C.  $b_1 = 1$  for a random walk without drift and  $b_1 \neq 1$  for a random walk with drift.

**LO.j: Describe implications of unit roots for time-series analysis, explain when unit roots are likely to occur and how to test for them, and demonstrate how a time series with a unit root can be transformed so it can be analyzed with an AR model.**

10. An analyst is checking for unit root problem and has performed the Dickey Fuller test. He found that the null ( $\rho=0$ ) cannot be rejected. Which of the following statements is *most accurate*?
- A. The time series does not have a unit root problem.
  - B. The time series is covariance stationary.
  - C. The time series has a unit root problem.

**LO.k: Describe the steps of the unit root test for nonstationarity and explain the relation of the test to autoregressive time-series models.**

11. If a time-series model has a unit root problem then which of the following transformations can *most likely* be performed to solve this problem?
- A. Log-linear transformation.
  - B. First-differencing.
  - C. Dickey Fuller transformation.

**LO.l: Explain how to test and correct for seasonality in a time-series model and calculate and interpret a forecasted value using an AR model with a seasonal lag.**

12. The following table shows the autoregression output for Log-quarterly sales of a retailer using an AR(1 model). The number of observations are 40. Which of the following statements is *most accurate*?

(At a significance level of 5% and 37 degrees of freedom the critical t-value is 2.026)

Residual Lag	t-statistic
1	-0.45
2	-0.02
3	0.015
4	4.815

- A. Seasonality is present in the time series.
- B. Seasonality is not present in the time series.
- C. The AR model is specified correctly.

**LO.m: Explain autoregressive conditional heteroskedasticity (ARCH) and describe how ARCH models can be applied to predict the variance of a time series.**

13. The regression results for a ARCH(1) model are shown below. If the current period squared error is 0.4356, the variance of the error terms in the next period is *closest* to:

	Coefficients	p-value
Constant	5.6521	<0.001
Lag 1	0.3496	<0.001

- A. 5.652.

- B. 5.804.
- C. 6.001.

**LO.n: Explain how time-series variables should be analyzed for nonstationarity and/or cointegration before use in a linear regression.**

14. Linear regression is *least appropriate* for modelling the relationship between two time series when:
- A. neither series has a unit root.
  - B. only one series has a unit root.
  - C. both series have a unit root and are cointegrated.

**LO.o: Determine an appropriate time-series model to analyze a given investment problem and justify that choice.**

## Solutions

1. B is correct. The predicted trend value of a time series in a log-linear trend model is  $e^{b_0+b_1t}$ . Therefore at  $t=20$  the predicted trend value is  $e^{10+5 \times 20} = 5.92$
2. A is correct. When the variable grows at a constant rate, the log-linear model is most appropriate. When the variable grows increases over time by a constant amount, a linear trend model is most appropriate. A Durbin Watson statistic significantly different from 2.0 suggests that the residual terms are correlated and it may be appropriate to use an autoregressive model.
3. A is correct. For a time series to be covariance stationary the expected value of the time series must be constant over time (this value is known as the mean-reverting level). Statements B and C are correct.
4. C is correct. One step ahead forecast =  $2.5 + 0.2 \times 4 = 3.3$   
Two step ahead forecast =  $2.5 + 0.2 \times 3.3 = 3.16$
5. C is correct. The standard error is  $1/\sqrt{144} = 0.08333$ .  
The t-stat for Lag 1 is  $0.124 / 0.08333 = 1.488$   
The t-stat for Lag 2 is  $0.148 / 0.08333 = 1.77$   
The t-stat for Lag 3 is  $0.166 / 0.08333 = 1.99$   
Since the t-stat for Lag 3 is more than the critical value of 1.98 we cannot say that autocorrelation of the residual for Lag 3 is statistically different from 0. Hence the model is not specified correctly.
6. B is correct. The mean reverting level can be computed as:  $b_0/(1 - b_1) = 2.5/(1 - 0.2) = 3.125$
7. C is correct. The model with the lower RMSE for the out-of-sample data will have lower forecast error and will be expected to have better predictive power in the future.
8. B is correct. The coefficients of models estimated with shorter time series are usually more stable than those with longer time series because a longer sample period increases the chances that the underlying economic process has changed.
9. C is correct. In either case (with or without a drift)  $b_1 = 1$ .
10. C is correct. If the null  $g=0$  cannot be rejected, then  $b_1 - 1 = 0$  and  $b_1 = 1$ . Therefore the series has a unit root problem.
11. B is correct. If we believe that a time series is a random walk (i.e. it has a unit root problem) then we can use first differencing to produce a covariance stationary time series.

12. A is correct. The t-stat for Lag 4 is higher than the critical t-value. Thus we can conclude that seasonality is present in the time series and the model is misspecified.
13. B is correct. Since the p-value for the coefficient on the lagged variable indicates statistical significance, we can conclude that the time series is ARCH(1). The variance of the error term in the next period can be computed as:  
 $5.6521 + 0.3496 (0.4356) = 5.804$ .
14. B is correct. If only one time series has a unit root, then we should not use linear regression.